

Concerning the CUSUM

A simple way to analyze a cumulative sum chart using Excel

by Vladimir Shper

IT IS WELL KNOWN that the cumulative sum (CUSUM) technique is one of the most widely used for data analysis.^{1,2} The technique is usually recommended for monitoring the changes in process characteristics because CUSUM charts “are much more effective than Shewhart charts in detecting small and moderate-sized sustained shifts in the parameters of the probability distribution of a quality characteristic.”³

Contrasted with simple Shewhart charts in which you visually analyze process stability—seeing whether there are points beyond the control limits—visual analysis of a CUSUM chart is rather tiresome. It requires that you construct a so-called V-mask with arms pointing to the left and a central point, which should be placed on each point of the process as it is plotted.

Due to such inconvenience, most textbooks recommend an algorithmic (or table) method to analyze the process stability while working with CUSUM charts.⁴ But there’s a simple procedure for anyone to plot a moving V-mask on a CUSUM graph by using traditional Excel software.

Example with Excel

Figure 1 shows a CUSUM chart constructed from data in Donald J. Wheeler’s *Advanced Topics in Statistical Process Control*.⁵ All the values of this curve are taken from column E (Figure 2). This is a typical graph calculated from the formula $S_i = S_{i-1} - T + x_i$, in which T denotes a target of the process. In this example, $T = 50$.⁶

Calculate the parameters of V-mask as it’s described in any guide. Again, using data from *Advanced Topics in Statistical Process Control*: $H = 4,876$ and $K = 1,84$,⁷ in which H is a critical distance of V-mask and K is its slope (Figure 3). Now, choose any cell in the column next to column E (for example, E15 in Figure 2) and construct two straight lines with the slope K starting at points deviated from the chosen point by H units.

In cell F15, enter the formula: =E15 - \$L\$30 (\$L\$30 contains the value of H). In cell G15, enter the formula: =E15 + \$L\$30. For cells F14 and G14, enter =E15 - \$L\$30 - \$L\$29 and =E15 + \$L\$30 + \$L\$29, respectively (\$L\$29 contains the value of K).

For cells F13 and G13, you have: =E15 - \$L\$30 - 2 * \$L\$29, and =E15 + \$L\$30 + 2 * \$L\$29, etc. As a result, you can obtain something like Figure 4. Here, crimson lines show the mask arms pointing to the left, and yellow lines for those pointing to the right (the values for these lines are shown in columns H and I of Figure 2).

To draw a V-mask, it is necessary to copy the set of cells that correspond to your lines (in this example, cells F11:G15) and place it into a cell of interest. Doing this, you should take into account the length of the V-mask arm or how many cells you used for depicting the sloping lines.

If the regions of two successive drawings overlap, erase the previous one. Otherwise, several extra lines will appear and make the picture a bit ambiguous. A comparison with data in *Advanced Topics in Statistical Process Control*⁸ shows that Figure 4 leads to the same results as a traditional V-mask.

Apparently, Excel allows you to easily change the parameters of V-mask. Such a simple technique may be useful for quality practitioners, especially when you consider the ubiquitous character of Excel application.

REFERENCES

1. Donald J. Wheeler, *Advanced Topics in Statistical Process Control*, SPC Press, 1995.
2. Douglas C. Montgomery, *Statistical Quality Control*, sixth edition, 2009.
3. William H. Woodall, "Controversies and Contradictions in Statistical Process Control," *Journal of Quality Technology*, 2000, Vol. 32, No. 4, pp. 341-350.
4. Montgomery, *Statistical Quality Control*, see reference 2.
5. Wheeler, *Advanced Topics in Statistical Process Control*, see reference 1, chapter 13, Table 13.1.
6. Wheeler, *Advanced Topics in Statistical Process Control*, see reference 1.
7. Wheeler, *Advanced Topics in Statistical Process Control*, see reference 1, p. 293.
8. Wheeler, *Advanced Topics in Statistical Process Control*, see reference 1, chapter 13, Figures 13.3-13.5.

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